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A SHEAF OF HOCHSCHILD COMPLEXES ON QUASI-COMPACT OPENS

WENDY LOWEN

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Abstract. For a scheme X, we construct a sheaf $\mathbf{C}$ of complexes on X such that for every quasi-compact open $U \subset X$, $\mathbf{C}(U)$ is quasi-isomorphic to the Hochschild complex of U (Lowen and Van den Bergh, 2005). Since $\mathbf{C}$ is moreover acyclic for taking sections on quasi-compact opens, we obtain a local to global spectral sequence for Hochschild cohomology if X is quasi-compact.

1. Introduction

Let X be a scheme over a field k. In [11], the Hochschild complex $\mathbf{C}(X, \mathcal{O}_X)$ of X is defined to be the Hochschild complex of the abelian category $\mathsf{Mod}(X)$ of sheaves on X. Its cohomology theory coincides with various notions of Hochschild cohomology of X considered in the literature, for example by Swan [14] and Kontsevich [8], which in the commutative case agree with the earlier theory of Gerstenhaber-Schack [2].

For a basis $\mathfrak{b}$ of affine opens of X, there is an associated k-linear category (also denoted by $\mathfrak{b}$) and there is a quasi-isomorphism

$$\mathbf{C}(X, \mathcal{O}_X) \cong \mathbf{C}(\mathfrak{b})$$

where $\mathbf{C}(\mathfrak{b})$ is the Hochschild complex of the k-linear category $\mathfrak{b}$ (§2.1). The Hochschild complexes have a considerable amount of extra structure containing in particular the cup-product and the Gerstenhaber bracket. This extra structure is important for deformation theory. It is captured by saying that the complexes are B_∞-algebras [3, 6], and $\cong$ as above means the existence of an isomorphism in the homotopy category of B_∞-algebras. Let $\mathcal{O}_{\mathfrak{b}}$ be the restriction of $\mathcal{O}_X$ to the basis $\mathfrak{b}$. Then $\cong$ above is reflected in the fact that there is an equivalence between the deformation theory of $\mathsf{Mod}(X)$ as an abelian category [12] and the deformation theory of $\mathcal{O}_{\mathfrak{b}}$ as a twisted presheaf [9].

If we consider the restrictions $\mathfrak{b}|_U$ of $\mathfrak{b}$ to open subsets $U \subset X$, we obtain a presheaf of Hochschild complexes on X:

$$\mathbf{C}_{\mathfrak{b}} : U \longmapsto \mathbf{C}(\mathfrak{b}|_U).$$

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To relate the "global" Hochschild complex $\mathbf{C}(\mathfrak{b})$ to the "local" Hochschild complexes $\mathbf{C}(\mathfrak{b}|_U)$ of certain open subsets $U \subset X$, it would be desirable for $\mathbf{C}_{\mathfrak{b}}$ to be a sheaf, which is preferably acyclic for taking global sections. Unfortunately, $\mathbf{C}_{\mathfrak{b}}$ is not even a separated presheaf with regard to finite coverings. In this paper, we construct a *sheaf* $\mathbf{C}$ of B_∞-algebras such that

(1) $\mathbf{C}(U, \mathcal{O}_U) \cong \mathbf{C}(U)$ *for* U *quasi-compact open.*
(2) $\mathbf{C}$ is acyclic for taking quasi-compact sections, i.e. $R\Gamma(U, \mathbf{C}) = \mathbf{C}(U)$ *for* U *quasi-compact open.*

For U quasi-compact open, $\mathbf{C}(U)$ is obtained as a colimit of complexes $\mathbf{C}_{\mathfrak{b}}(U)$ over a collection $\mathcal{B}(U)$ of bases of U (§2.3). The properties of $\mathbf{C}$ depend upon the choice of a *good* presheaf $\mathcal{B}$ of bases (Definition 2.1).

From properties (1) and (2), we readily deduce the existence of a local to global spectral sequence

$$E_2^{p,q} = H^p(X, \mathbf{H}^q\mathbf{C}) \Rightarrow H^{p+q}\mathbf{C}(X)$$

for Hochschild cohomology for a quasi-compact scheme X (Theorem 4.1).

We should remark that for a smooth separated scheme, another sheaf of B_∞-algebras $\mathbf{D}_{\mathrm{poly}}$ is considered, for example, by Kontsevich [7], Van den Bergh [15], and Yekutieli [16]. Let $\mathbf{C}(\mathcal{O}(U))$ be the Hochschild complex of the ring $\mathcal{O}(U)$, and let $\mathbf{C}_{\mathrm{poly}}(\mathcal{O}(U))$ be the subcomplex which consists of the polydifferential operators, i.e. multilinear maps $\mathcal{O}(U)^{\otimes p} \longrightarrow \mathcal{O}(U)$ which are differential operators in each argument. Then for U *affine* open, $\mathbf{D}_{\mathrm{poly}}$ satisfies

$$\mathbf{D}_{\mathrm{poly}}(U) \cong \mathbf{C}_{\mathrm{poly}}(\mathcal{O}(U)).$$

The complex $R\Gamma(X, \mathbf{D}_{\mathrm{poly}})$ computes the Hochschild cohomology of X, but a priori does not inherit the structure of a B_∞-algebra. One way to overcome this problem is by using a fibrant resolution $\mathbf{D}_{\mathrm{poly}} \longrightarrow \mathbf{F}_{\mathrm{poly}}$ in the model category of presheaves of B_∞-algebras as defined by Hinich [5]. Alternatively, in [15, Appendix B], Van den Bergh constructs a quasi-isomorphic object $R\Gamma(X, \mathbf{D}_{\mathrm{poly}})^{\mathrm{tot}}$ that *does* inherit this structure (the construction, which uses pro-hypercoverings, is functorial and inherits any operad-algebra structure). Moreover, $R\Gamma(X, \mathbf{D}_{\mathrm{poly}})^{\mathrm{tot}}$ is isomorphic to $\mathbf{C}(X, \mathcal{O}_X)$ in the homotopy category of B_∞-algebras [15, Theorem 3.1, Appendices A, B] and by [15, Appendix B.10], we actually have $R\Gamma(X, \mathbf{D}_{\mathrm{poly}})^{\mathrm{tot}} \cong \Gamma(X, \mathbf{F}_{\mathrm{poly}})$ in the same sense.

Finally, as to the existence of a local to global spectral sequence for Hochschild cohomology for a general ringed space $(X, \mathcal{O}_X)$, a proof using hypercoverings is in preparation [10].

2. Presheaves of Hochschild complexes

2.1. **The Hochschild complex of a scheme.** Throughout, k is a field. Let $(X, \mathcal{O}_X)$ be a scheme over k and let $\mathfrak{b}$ be a basis of affine opens. We use the same notation for the associated k-linear category with $\mathfrak{b}$ as objects and

$$\mathfrak{b}(V, U) = \begin{cases} \mathcal{O}_X(V) & \text{if } V \subset U, \\ 0 & \text{else.} \end{cases}$$

In [11, §7.1], the Hochschild complex $\mathbf{C}(X, \mathcal{O}_X)$ of X is defined, and in [11, Theorem 7.3.1], there is shown to be a quasi-isomorphism

$$\mathbf{C}(X, \mathcal{O}_X) \cong \mathbf{C}(\mathfrak{b}),$$

where $\mathbf{C}(\mathfrak{b})$ is the Hochschild complex of the k-linear category $\mathfrak{b}$ [13], i.e.

$$\mathbf{C}^p(\mathfrak{b}) = \prod_{U_0,\dots,U_p\in\mathfrak{b}} \mathrm{Hom}_k(\mathfrak{b}(U_{p-1},U_p)\otimes_k \cdots \otimes_k \mathfrak{b}(U_0,U_1), \mathfrak{b}(U_0,U_p)),$$

and the differential is the usual Hochschild differential. More concretely, we have

$$\mathbf{C}^p(\mathfrak{b}) = \prod_{U_0\subset U_1\subset\cdots\subset U_p\in\mathfrak{b}} \mathrm{Hom}_k(\mathcal{O}_X(U_{p-1})\otimes_k \cdots \otimes_k \mathcal{O}_X(U_0), \mathcal{O}_X(U_0)),$$

$$\mathbf{C}^0(\mathfrak{b}) = \prod_{U_0\in\mathfrak{b}} \mathcal{O}_X(U_0).$$

Hence this complex combines the nerve of the poset $\mathfrak{b}$ with the algebraic structure of $\mathcal{O}_X$. In fact, both complexes are B_∞-algebras [3, 6], and $\cong$ means the existence of an isomorphism in the homotopy category of B_∞-algebras.

2.2. **The presheaf $\mathbf{C}_{\mathcal{B}}$ of Hochschild complexes.** For an arbitrary open subset $U \subset X$, put $\mathfrak{b}|_U = \{B \in \mathfrak{b} \mid B \subset U\}$. Then $\mathfrak{b}|_U$ is a basis of affine opens for the scheme $(U, \mathcal{O}_U)$; hence we have a quasi-isomorphism

$$\mathbf{C}(U,\mathcal{O}_U) \cong \mathbf{C}(\mathfrak{b}|_U).$$

For $V \subset U$ there is an obvious restriction map

$$\mathbf{C}(\mathfrak{b}|_U) \longrightarrow \mathbf{C}(\mathfrak{b}|_V).$$

We thus obtain a presheaf

$$\mathbf{C}_{\mathfrak{b}} : U \longmapsto \mathbf{C}_{\mathfrak{b}}(U) = \mathbf{C}(\mathfrak{b}|_U)$$

of Hochschild complexes on X. It is readily seen that in general, $\mathbf{C}_{\mathfrak{b}}$ fails to be a sheaf. Indeed, suppose we have $W \in \mathfrak{b}$ and $W = U \cup V$ with U and V proper open subsets. Then there is a non-zero element $\varphi = (\varphi_{U_0})_{U_0} \in \mathbf{C}^0_{\mathfrak{b}}(W)$ with

$$\varphi_{U_0} = \begin{cases} 1 \in \mathcal{O}_X(U_0) & \text{if } U_0 = W, \\ 0 & \text{else,} \end{cases}$$

whose restriction to U and V is zero. In this example, the fact that $W = U \cup V$ makes the presence of W in $\mathfrak{b}$ redundant. This suggests that to obtain a sheaf, we must work with variable bases, as will be done in the next section.

2.3. **The presheaf $\mathbf{C}_{\mathcal{B}}$ of colimit Hochschild complexes.** In this section instead of considering $\mathbf{C}_{\mathfrak{b}}(U)$ for a fixed basis $\mathfrak{b}$ of X, we will consider a colimit of complexes $\mathbf{C}(\mathfrak{b})$ over different bases $\mathfrak{b}$ of U. More precisely, we are looking for collections $\mathcal{B}(U)$ of bases of affine opens of U, which allow us to define "colimit Hochschild complexes"

$$\mathbf{C}_{\mathcal{B}}(U) = \mathrm{colim}_{\mathfrak{b}\in\mathcal{B}(U)} \mathbf{C}(\mathfrak{b}).$$

Here $\mathcal{B}(U)$ is ordered by $\supset$ and $\mathfrak{b} \supset \mathfrak{b}'$ corresponds to the canonical $\mathbf{C}(\mathfrak{b}) \longrightarrow \mathbf{C}(\mathfrak{b}')$. Since we do not want the colimit to change the cohomology, we want it to be a filtered colimit. In particular, this is the case if $\mathcal{B}(U)$ is closed under intersections, i.e. if we have the operation

$$\mathcal{B}(U) \times \mathcal{B}(U) \longrightarrow \mathcal{B}(U) : (\mathfrak{b}, \mathfrak{b}') \longmapsto \mathfrak{b} \cap \mathfrak{b}'. \tag{1}$$

Note that in general, $\mathfrak{b} \cap \mathfrak{b}'$ need not even be a basis. If $\mathcal{B}(U) \neq \varnothing$ and we have (1), then there are quasi-isomorphisms

$$\mathbf{C}(U,\mathcal{O}_U) \cong \mathbf{C}_{\mathcal{B}}(U).$$

For $\mathbf{C}_{\mathcal{B}} : U \longmapsto \mathbf{C}_{\mathcal{B}}(U)$ to become a presheaf, we need restriction operations

$$\mathcal{B}(U) \longrightarrow \mathcal{B}(V) : \mathfrak{b} \longmapsto \mathfrak{b}|_V = \{B \in \mathfrak{b} \,|\, B \subset V\} \tag{2}$$

for $V \subset U$, making $\mathcal{B}$ itself into a presheaf of collections of bases. In this way, $\mathbf{C}_{\mathcal{B}}$ clearly becomes a presheaf of B_∞-algebras on X.

In order to prove Proposition 3.1 in the next section, we need two more operations on $\mathcal{B}$. First, we want to take the union of bases coinciding on the intersection of their domains; i.e. we want the operation

$$\mathcal{B}(U) \times_{\mathcal{B}(U \cap V)} \mathcal{B}(V) \longrightarrow \mathcal{B}(U \cup V) : (\mathfrak{b}, \mathfrak{b}') \longmapsto \mathfrak{b} \cup \mathfrak{b}'. \tag{3}$$

Second, we want to refine bases by plugging in finer bases; i.e. for $V \subset U$ we want the operation

$$\mathcal{B}(U) \times \mathcal{B}(V) \longrightarrow \mathcal{B}(U) : (\mathfrak{b}_U, \mathfrak{b}_V) \longmapsto \mathfrak{b}_U \circ \mathfrak{b}_V = \{B \in \mathfrak{b}_U \,|\, B \subset V \implies B \in \mathfrak{b}_V\}. \tag{4}$$

Note that (1) is just a special case of (4). Also, combining (2), (3) and (4) yields the following refinement operation on $\mathcal{B}$. If δ is any finite collection of open subsets of U (not necessarily covering U), we have

$$\mathcal{B}(U) \longrightarrow \mathcal{B}(U) : \mathfrak{b} \longmapsto \mathfrak{b}_\delta = \{B \in \mathfrak{b} \,|\, V \subset \cup\delta \implies \exists D \in \delta, V \subset D\}. \tag{5}$$

Definition 2.1. $\mathcal{B}$ is called *good* if $\mathcal{B}(X) \neq \varnothing$ and $\mathcal{B}$ has the operations (1),…, (5).

We will now show that there exists a good $\mathcal{B}$.

Proposition 2.2.
(1) *If $\mathcal{B}$ with $\mathcal{B}(X) \neq \varnothing$ has* (2), (3) *and* (4)*, then it is good.*
(2) *Let $\mathfrak{b}$ be any basis of affine opens of X. There exists a smallest good $\mathcal{B}$ with $\mathfrak{b} \in \mathcal{B}(X)$. This $\mathcal{B}$ is given by*

$$\mathcal{B}(U) = \{(\mathfrak{b}|_U)_{\delta_1,\dots,\delta_n} \,|\, \delta_i \subset \mathrm{open}(U),\, |\delta_i| < \infty\}.$$

Proof. (1) follows from the discussion above. For (2), first note that $\mathcal{B}$ is obviously contained in any good $\mathcal{B}'$. If $V \subset U$ and δ is a collection in U, we put $\delta|_V = \{D \cap V \,|\, D \in \delta\}$. For any basis $\mathfrak{b}'$ of U, we have $(\mathfrak{b}'_\delta)|_V = (\mathfrak{b}'|_V)_{(\delta|_V)}$, so (2) holds. For (4), note that $(\mathfrak{b}|_U)_{\delta_1,\dots,\delta_n} \circ (\mathfrak{b}|_V)_{\varepsilon_1,\dots,\varepsilon_m} = (B|_U)_{\delta_1,\dots,\delta_n,\varepsilon_1,\dots,\varepsilon_m}$. Finally for (3), if $(\mathfrak{b}|_U)_{\delta_1,\dots,\delta_n}$ and $(\mathfrak{b}|_V)_{\varepsilon_1,\dots,\varepsilon_m}$ coincide on $U \cap V$, then their union equals $(\mathfrak{b}|_{U\cup V})_{\delta_1,\dots,\delta_n,\varepsilon_1,\dots,\varepsilon_m,\{U,V\}}$. □

3. Sheaves of Hochschild complexes

3.1. **The presheaf $\mathbf{C}_{\mathcal{B}}$ for a good $\mathcal{B}$.** From now on, $\mathcal{B}$ is a good presheaf of bases, and we consider the presheaf $\mathbf{C}_{\mathcal{B}}$ of colimit Hochschild complexes as defined in §2.3.

Proposition 3.1.
(1) *$\mathbf{C}_{\mathcal{B}}$ is flabby.*
(2) *$\mathbf{C}_{\mathcal{B}}$ satisfies the sheaf condition with respect to finite coverings.*

Proof. (2) By induction, we may consider $U = U_1 \cup U_2$ and the given elements $\varphi_i \in \mathbf{C}^p_{\mathcal{B}}(U_i)$ such that $\varphi_1|_{U_{12}} = \varphi_2|_{U_{12}}$, where $U_{12} = U_1 \cap U_2$. Let φ_i be a representing element in $\mathbf{C}^p(\mathfrak{b}_i)$ for a basis $\mathfrak{b}_i \in \mathcal{B}(U_i)$ and let $\mathfrak{b}' \subset \mathfrak{b}_1|_{U_{12}} \cap \mathfrak{b}_2|_{U_{12}}$ be a basis in $\mathcal{B}(U_{12})$ for which $\varphi_1|_{U_{12}}$ and $\varphi_2|_{U_{12}}$ coincide in $\mathbf{C}^p(\mathfrak{b}')$. Put $\mathfrak{b}'_i = \mathfrak{b}_i \circ \mathfrak{b}' \in \mathcal{B}(U_i)$ (using (4)) and put $\mathfrak{b} = \mathfrak{b}'_1 \cup \mathfrak{b}'_2 \in \mathcal{B}(U \cup V)$ (using (3)). We can now easily give an element $\varphi \in \mathbf{C}^p(\mathfrak{b})$, which represents a glueing of φ_1 and φ_2 on U, by specifying its value for $V_0 \subset \cdots \subset V_p$: if $V_p \in \mathfrak{b}'_i$, we use the element specified by φ_i. This is well defined since $V_p \in \mathfrak{b}'_1 \cap \mathfrak{b}'_2$ implies $V_p \in \mathfrak{b}'$. It is a glueing of the φ_i since φ and φ_i coincide on $\mathfrak{b}'_i \subset \mathfrak{b}_i$.

Now suppose we have an element $\varphi \in \mathbf{C}^p(\mathfrak{b}')$ for $\mathfrak{b}' \in \mathcal{B}(U)$ and suppose we have bases $\mathfrak{b}_i \subset \mathfrak{b}'|_{U_i}$ for which $\varphi|_{U_i}$ becomes zero in $\mathbf{C}^p(\mathfrak{b}_i)$. If we put $\mathfrak{b}'_i = \mathfrak{b}_i \circ (\mathfrak{b}_1|_{U_{12}} \cap \mathfrak{b}_2|_{U_{12}})$ and $\mathfrak{b} = \mathfrak{b}'_1 \cup \mathfrak{b}'_2$, then φ becomes zero in $\mathbf{C}^p(\mathfrak{b}')$.

(1) Consider the restriction map $\mathbf{C}^p(U) \longrightarrow \mathbf{C}^p(V)$ for $V \subset U$. If $\varphi \in \mathbf{C}^p(\mathfrak{b})$ is a representing element in the codomain, we can lift it to $\bar{\varphi} \in \mathbf{C}^p(\mathfrak{b}' \circ \mathfrak{b})$, where $\mathfrak{b}' \in \mathcal{B}(U)$ is arbitrary and the value of $\bar{\varphi}$ for $V_0 \subset \cdots \subset V_p$ is the value specified by φ if $V_p \subset V$ and is zero otherwise. □

3.2. **The sheaf $\mathbf{C}_{\mathrm{qc}}$ of colimit Hochschild complexes.** Let $\mathrm{qc}(X) \subset \mathrm{open}(X)$ be the subposet of quasi-compact opens with the induced Grothendieck topology. We immediately get:

Proposition 3.2. *The restriction $\mathbf{C}_{\mathrm{qc}}$ of $\mathbf{C}_{\mathcal{B}}$ to $\mathrm{qc}(X)$ is a sheaf.*

Proof. Since every covering of a quasi-compact $U \subset X$ has a finite subcovering, it suffices to check the sheaf condition on finite coverings, which is done in Proposition 3.1. □

3.3. **The sheaf $\mathbf{C} = \mathbf{C}_{\mathcal{B}}$.** Let $\mathsf{Pr}(X)$ and $\mathsf{Sh}(X)$ (resp. $\mathsf{Pr}_{\mathrm{qc}}(X)$ and $\mathsf{Sh}_{\mathrm{qc}}(X)$) be the categories of presheaves and sheaves on X (resp. on $\mathrm{qc}(X)$). Since $\mathrm{qc}(X)$ is a basis of X, by the (proof of the) Lemme de Comparaison [1], there is a commutative square

$$\begin{tikzcd} \mathsf{Pr}(X) \arrow[r, "(-)_{\mathrm{qc}}"] \arrow[d, "a"'] & \mathsf{Pr}_{\mathrm{qc}}(X) \arrow[d, "a'"] \\ \mathsf{Sh}(X) \arrow[r, "(-)_{\mathrm{qc}}", "\cong"'] & \mathsf{Sh}_{\mathrm{qc}}(X) \end{tikzcd}$$

in which the vertical arrows are sheafifications, the horizontal arrows are restrictions to $\mathrm{qc}(X)$, and the lower horizontal arrow is an equivalence. Let $\mathbf{C} = a\mathbf{C}_{\mathcal{B}}$ be the sheafification of $\mathbf{C}_{\mathcal{B}}$.

Proposition 3.3. *If $U \subset X$ is a quasi-compact open, then*

$$\mathbf{C}_{\mathcal{B}}(U) \longrightarrow \mathbf{C}(U)$$

is an isomorphism. In particular, there is a quasi-isomorphism

$$\mathbf{C}(U, \mathcal{O}_U) \cong \mathbf{C}(U).$$

Proof. By Proposition 3.2 we have $(\mathbf{C}_{\mathcal{B}})_{\mathrm{qc}} \cong a'(\mathbf{C}_{\mathcal{B}})_{\mathrm{qc}} \cong (a\mathbf{C}_{\mathcal{B}})_{\mathrm{qc}}$. □

Proposition 3.4. *$\mathbf{C}^p$ is acyclic for taking quasi-compact sections; i.e. for $U \subset X$ a quasi-compact open and $i > 0$, we have $H^i(U, \mathbf{C}^p) = 0$.*

Proof. By Propositions 3.1(1) and 3.3, the restriction maps $\mathbf{C}^p(X) \longrightarrow \mathbf{C}^p(U)$ are surjective for U quasi-compact. The rest of the proof is along the lines of the classical proof that flabby sheaves are acyclic for taking global sections. □

4. Local to global spectral sequence

In this section, X is a quasi-compact scheme and $\mathbf{C}$ is the sheaf of complexes of §3.3. In particular, there are quasi-isomorphisms $\mathbf{C}(U, \mathcal{O}_U) \cong \mathbf{C}(U)$ for U quasi-compact open. We obtain a local to global spectral sequence for Hochschild cohomology:

Theorem 4.1. *There is a local to global spectral sequence*

$$E_2^{p,q} = H^p(X, \mathbf{H}^q\mathbf{C}) \Rightarrow H^{p+q}\mathbf{C}(X).$$

Proof. Since, by Proposition 3.4, $\mathbf{C}$ is a bounded below complex of acyclic objects for Γ, $\mathbf{C}$ is itself acyclic, i.e. $R\Gamma(X, \mathbf{C}) = \mathbf{C}(X)$. So the above is just the hypercohomology spectral sequence [4, 2.4.2] for the complex of sheaves $\mathbf{C}$. □

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Departement DWIS, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, Belgium
E-mail address: `wlowen@vub.ac.be`